Chapter 3

Three-phase half-wave AC voltage controllers

(Part 2)

By

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Analysis of three-phase half-wave AC voltage controller

Analysis of three-phase half-wave AC voltage controller with resistive load

Expressions for the instantaneous output phase voltage:

For α = 45°, the waveform of output phase voltage (v_{an})

instantaneous input voltage per phase

$$v_{AN} = \sqrt{2}V_s \sin \omega t$$

$$v_{BN} = \sqrt{2}V_s \sin\left(\omega t - \frac{2\pi}{3}\right)$$

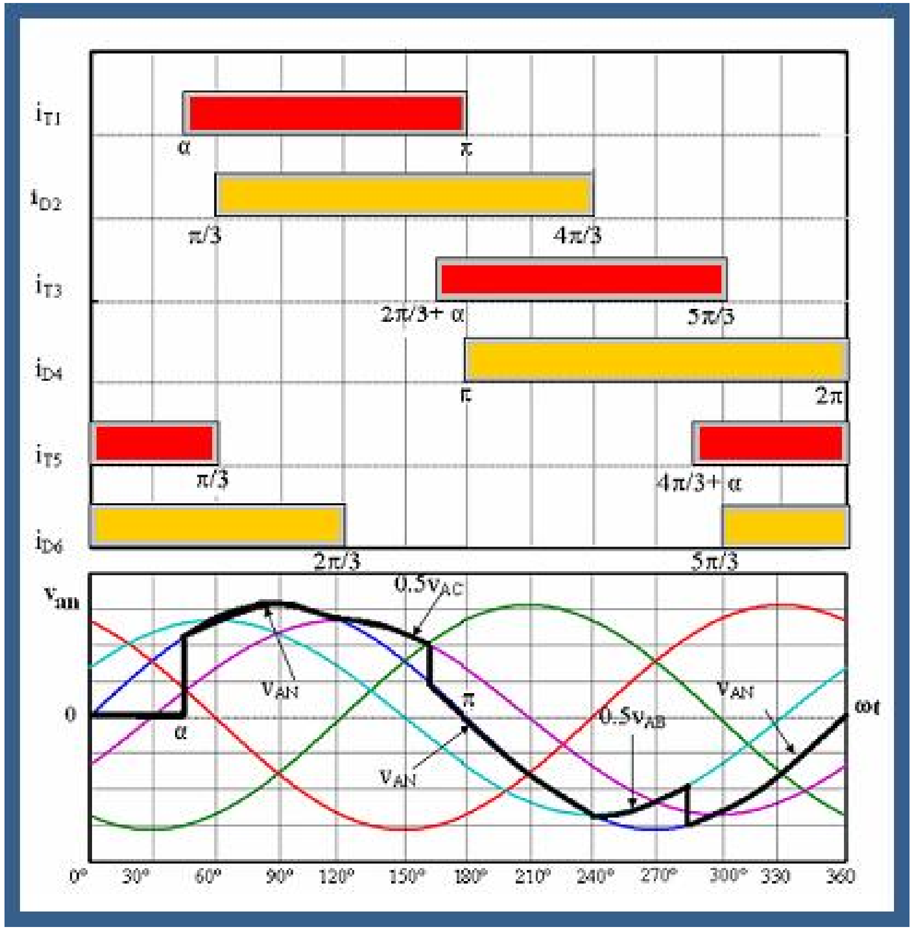
$$v_{CN} = \sqrt{2}V_s \sin\left(\omega t + \frac{2\pi}{3}\right)$$

instantaneous input line voltages

$$v_{AB} = \sqrt{6}V_s \sin\left(\omega t + \frac{\pi}{6}\right)$$

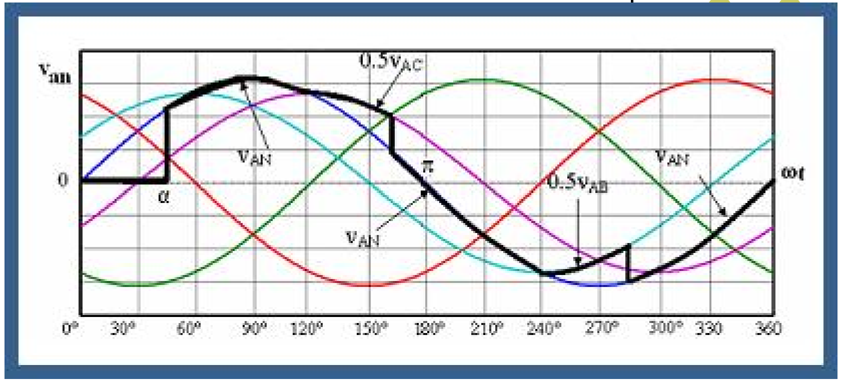
$$v_{BC} = \sqrt{6}V_s \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{CA} = \sqrt{6}V_s \sin\left(\omega t - \frac{7\pi}{6}\right)$$



Analysis of three-phase half-wave AC voltage controller with resistive load

For $\alpha = 45°$, the expressions for instantaneous output phase voltage (v_{an}) is given by:



Interval		Output
$0 \le \omega t < \alpha$	⇨	$v_{an} = 0$
$\alpha \le \omega t < 2\pi/3$	⇨	$v_{an} = v_{AN}$
$2\pi/3 \le \omega t < \alpha + 2\pi/3$	⇨	$v_{an} = 0.5\, v_{AC}$
$\alpha + 2\pi/3 \le \omega t < \pi$	⇨	$v_{an} = v_{AN}$
$\pi \le \omega t < 4\pi/3$	⇨	$v_{an} = v_{AN}$
$4\pi/3 \le \omega t < 4\pi/3 + \alpha$	⇨	$v_{an} = 0.5\, v_{AB}$
$\alpha + 4\pi/3 \le \omega t < 2\pi$	⇨	$v_{an} = v_{AN}$

$$v_{CA} = \sqrt{6}\, Vs \sin(\omega t - 7\pi/6)$$

$$v_{AC} = - v_{CA} = \sqrt{6}\, Vs \sin(\omega t - (7\pi/6 - \pi))$$

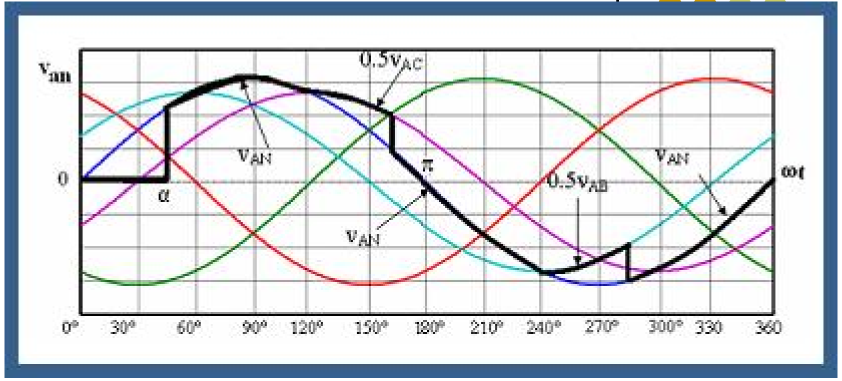
$$v_{AC} = \sqrt{6}\, Vs \sin(\omega t - \pi/6)$$

Analysis of three-phase half-wave AC voltage controller with resistive load

The rms value of the output phase voltage

$V_{an(rms)}$

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 \, d(\omega t)}$$



$$v_{an} = \left\{ \frac{1}{2\pi}\left[\int_{\alpha}^{2\pi/3} v_{AN}^2 . d\omega t + \int_{2\pi/3}^{2\pi/3+\alpha} (0.5 v_{AC})^2 d\omega t + \int_{2\pi/3+\alpha}^{\pi} v_{AN}^2 . d\omega t + \int_{\pi}^{4\pi/3} v_{AN}^2 . d\omega t + \int_{4\pi/3}^{4\pi/3+\alpha} (0.5 v_{AB})^2 d\omega t + \int_{4\pi/3+\alpha}^{2\pi} v_{AN}^2 . d\omega t \right] \right\}^{0.5}$$

$\alpha = 45°$

$v_{AN} = \sqrt{2}\ \text{Vs} \sin \omega t$

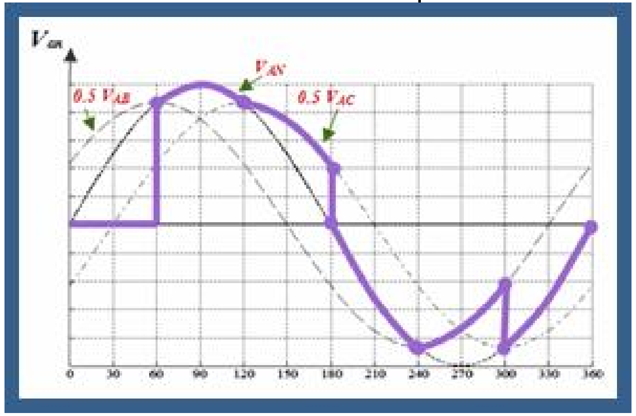
$v_{AB} = \sqrt{6}\ \text{Vs} \sin(\omega t + \pi/6)$

$v_{AC} = \sqrt{6}\ \text{Vs} \sin(\omega t - \pi/6)$

$$v_{an} = \sqrt{6}\ V_s \left(\frac{1}{2\pi}\left\{ \int_{\pi/4}^{2\pi/3} \left(\frac{1}{3}\right) \sin^2 \omega t . d(\omega t) + \int_{\pi/2}^{3\pi/4} \left(\frac{1}{4}\right) \sin^2 \omega t . d(\omega t) + \int_{11\pi/12}^{\pi} \left(\frac{1}{3}\right) \sin^2 \omega t . d(\omega t) + \int_{\pi}^{4\pi/3} \left(\frac{1}{3}\right) \sin^2 \omega t . d(\omega t) + \int_{3\pi/2}^{7\pi/4} \left(\frac{1}{4}\right) \sin^2 \omega t . d(\omega t) + \int_{19\pi/12}^{2\pi} \left(\frac{1}{3}\right) \sin^2 \omega t . d(\omega t) \right\} \right)^{0.5}$$

$$V_0 = \sqrt{3} V_s \sqrt{\frac{1}{\pi}\left(\frac{\pi}{3} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)}$$

Analysis of three-phase half-wave AC voltage controller with resistive load



RMS value of the output voltage

Firing angle range

$$0 \le \alpha < 60^o$$

- **Two thyristors and one diode conduct.**
- **One thyristor and one diode conduct.**
- **One thyristor and two diodes conduct.**

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 d(\omega t)}$$

$$60^o \le \alpha < 120^o$$

- **One Thyristor and one Diode conduct**
- **One Thyristor and two Diodes conduct**

$$60^o \le \alpha < 90^o$$

$$90^o \le \alpha < 120^o$$

$$120^o \le \alpha < 210^o$$

- **One Thyristor and one Diode conduct**

Analysis of three-phase half-wave AC voltage controller with resistive load

RMS value of the output voltage

- The expressions of the rms value of the output voltage per phase for balanced star-connected resistive load are :

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 \, d(\omega t)}$$

Mode (1)

$0^o \le \alpha < 90^o$ ⇒

$$V_0 = \sqrt{3}V_s\sqrt{\frac{1}{\pi}\left(\frac{\pi}{3}-\frac{\alpha}{4}+\frac{\sin 2\alpha}{8}\right)}$$

Mode (2)

$90^o \le \alpha < 120^o$ ⇒

$$V_0 = \sqrt{3}V_s\sqrt{\frac{1}{\pi}\left(\frac{11\pi}{24}-\frac{\alpha}{2}\right)}$$

Mode (3)

$120^o \le \alpha < 210^o$ ⇒

$$V_0 = \sqrt{3}V_s\sqrt{\frac{1}{\pi}\left(\frac{7\pi}{24}-\frac{\alpha}{4}+\frac{\sin 2\alpha}{16}-\frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

Analysis of three-phase half-wave AC voltage controller with resistive load

RMS Output current

$$I_0 = \frac{V_0}{R_L}$$

Where: $I_o = I_a$ is the Output (load) current in phase (a)

Output power

$$P_0 = 3 I_0^2 R_L$$

Input VA rating

$$VA = 3 V_s I_o$$

Supply power factor

$$PF = \frac{P_0}{VA}$$

Ex .1: The three-phase unidirectional controller supplies a wye-connected resistive load of 10 Ω/phase and the line-to-line input voltage is 380-V, 50 Hz. The firing delay angle is 150°.
(a) Draw the output phase voltage waveform.
(b) Drive the output phase voltage expression.
(b) Determine the rms output phase voltage and current.
(b) Determine the input power factor.

Solution

$V_L = 380v \qquad f_s = 50\ Hz \qquad R = 10\ ohm \qquad \alpha = 150^o$

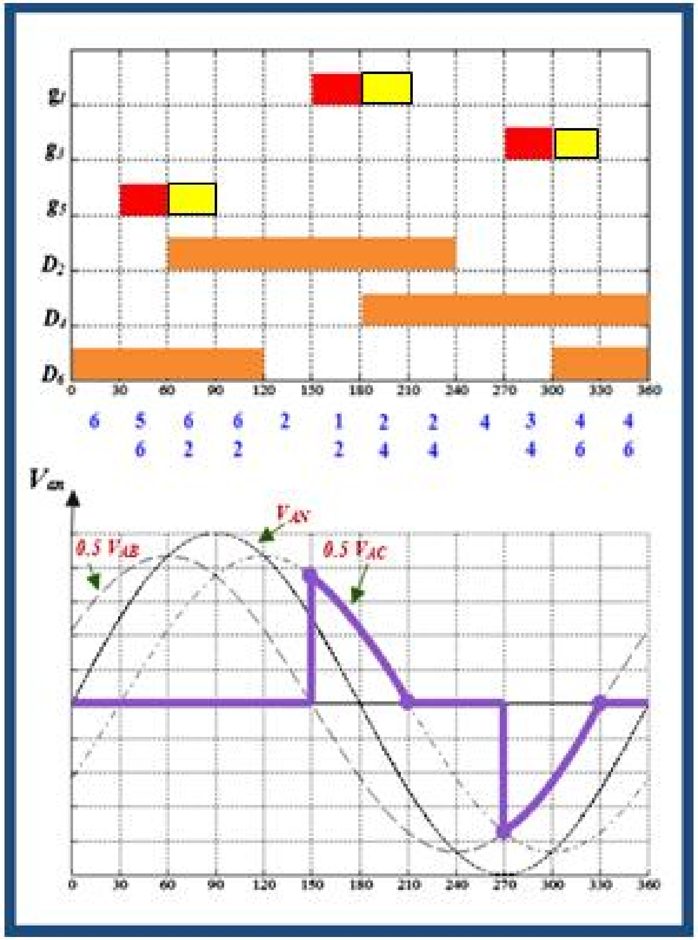
(a) the rms output phase voltage

Mode (3)
Firing angle range
(120° £ α < 210°)

$$V_0 = \sqrt{3}V_s\sqrt{\frac{1}{\pi}\left(\frac{7\pi}{24} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

$V_{an(rms)}$

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2\, d(\omega t)}$$



$$V_{a(rms)} = \left(\frac{1}{2\pi}\left[\int_{5\pi/6}^{7\pi/6} (0.5\, v_{AC})^2 . d\omega + \int_{3\pi/2}^{11\pi/6} (0.5\, v_{AB})^2 . d\omega\right]\right)^{0.5}$$

$$v_{AB} = \sqrt{6}\, V_s \sin(\omega t + \pi/6)$$

$$v_{AC} = \sqrt{6}\, V_s \sin(\omega t - \pi/6)$$

$$V_{a(rms)} = \sqrt{6} V_s \left(\frac{1}{2\pi}\left[\int_{2\pi/3}^{\pi} \left(\frac{1}{4}\right)\sin^2\omega t.d\omega t + \int_{4\pi/3}^{5\pi/3} \left(\frac{1}{4}\right)\sin^2\omega t.d\omega t\right]\right)^{0.5}$$

$$V_0 = \sqrt{3} V_s \sqrt{\frac{1}{\pi}\left(\frac{7\pi}{24} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

$$V_s = 380/\sqrt{3} = 219.4\text{v}$$

$$V_{a(rms)} = \sqrt{3}\ \text{x}219.4 \left(\frac{1}{\pi}\left\{\frac{7\pi}{24} - \frac{150\,\pi}{4\text{x}180} + \frac{\sin 2\text{x}150}{16} + \frac{\sqrt{3}\cos 2\text{x}150}{16}\right\}\right)^{0.5} = 109.8\ \text{v}$$

RMS Output current

$$I_0 = \frac{V_0}{R_L}$$ ⇒ $I_o = 109.8/10 = 10.98\ A \approx 11\ A$

Output power

$$P_0 = 3 I_0^2 R_L$$ ⇒ $P_o = 3\ (11)^2 \times 10 = 3630\ w$

Input VA rating

$$VA = 3 V_s I_o$$ ⇒ $VA = 3 \times 219.4 \times 11 = 7240.2\ VA$

Supply power factor

$$PF = \frac{P_0}{VA}$$ ⇒ $PF = 3630 / 7240.2 = 0.5$ (lagging)

Ex .2: **The three-phase unidirectional AC voltage controller supplies a wye connected resistive load of 50 Ω, and the line to line voltage is 208V at 60Hz. The delay is α = π/3.**

(a) Draw the output phase voltage waveform.

(b) Drive the output phase voltage expression.

(c) Determine the rms value of output voltage and current

(d) Determine the input power factor.

(d) the expressions for the instantaneous output voltage of phase a.

Solution

$V_L = 208$ v $f_s = 60$ Hz R = 50 ohm $\alpha = 60° = \pi/3$ radians

(a) the rms output phase voltage

Mode (1)
Firing angle range
(0° £ α < 90°)

$$V_0 = \sqrt{3}V_s\sqrt{\frac{1}{\pi}\left(\frac{\pi}{3}-\frac{\alpha}{4}+\frac{\sin 2\alpha}{8}\right)}$$

$V_s = 208/\sqrt{3} = 120$ v

$V_{an(rms)}$

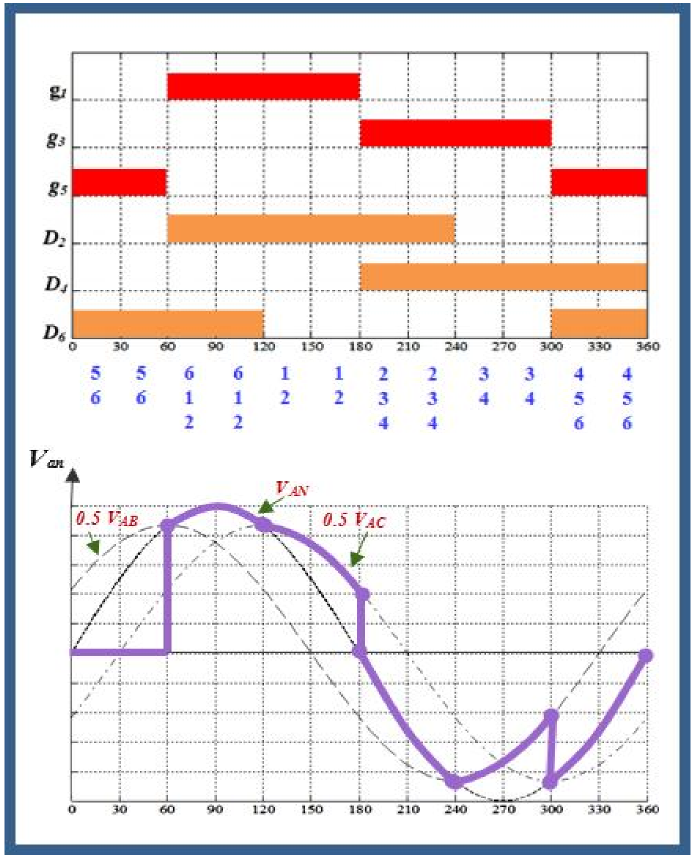
$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 \, d(\omega t)}$$

$$v_{an} = \left\{ \frac{1}{2\pi}\left[\int_{\pi/3}^{2\pi/3} v_{AN}^2 \, d\omega t + \int_{2\pi/3}^{\pi} (0.5 v_{AC})^2 . d\omega t + \int_{\pi}^{4\pi/3} v_{AN}^2 \, d\omega t + \int_{4\pi/3}^{5\pi/3} (0.5 v_{AB})^2 . d\omega t + \int_{5\pi/3}^{2\pi} v_{AN}^2 . d\omega t \right] \right\}^{0.5}$$

$$v_{AN} = \sqrt{2}\, V_s \sin \omega t$$

$$v_{AC} = \sqrt{6}\, V_s \sin(\omega t - \pi/6)$$

$$v_{AB} = \sqrt{6}\, V_s \sin(\omega t + \pi/6)$$



$$v_{an} = \sqrt{6}\, V_s \left(\frac{1}{2\pi}\left\{ \int_{\pi/3}^{2\pi/3} \left(\frac{1}{3}\right)\sin^2\omega t.d(\omega t) + \int_{\pi/2}^{5\pi/6} \left(\frac{1}{4}\right)\sin^2\omega t.d(\omega t) + \int_{\pi}^{4\pi/3} \left(\frac{1}{3}\right)\sin^2\omega t.d(\omega t) + \int_{3\pi/2}^{11\pi/6} \left(\frac{1}{3}\right)\sin^2\omega t.d(\omega t) + \int_{5\pi/3}^{2\pi} \left(\frac{1}{3}\right)\sin^2\omega t.d(\omega t) \right\} \right)^{0.5}$$

$$V_0 = \sqrt{3}\, V_s \sqrt{\frac{1}{\pi}\left(\frac{\pi}{3} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)}$$

$$V_{a(rms)} = \sqrt{3} \times 120 \left(\frac{1}{\pi} \left\{ \frac{\pi}{3} - \frac{\pi/3}{4} + \frac{\sin 2 \times 60^o}{8} \right\} \right)^{0.5} = 110.86 \text{ v}$$

RMS Output current

$$I_0 = \frac{V_0}{R_L}$$

I_o = 110.86/50 = 2.22 A

Output power

$$P_0 = 3 I_0^2 R_L$$

$P_o = 3\ (2.22)^2 \times 50$ = 739.26 w

Input VA rating

$$VA = 3 V_s I_o$$

VA = 3 x 120 x 2.22 = 799.2 VA

Supply power factor

$$PF = \frac{P_0}{VA}$$

PF = 739.26 / 799.2 = 0.925 (lagging)

Expressions for the instantaneous output voltage of phase a.

$V_s = 120$ v

$v_{AN} = \sqrt{2}\, Vs \sin\omega t$

For $0 \le \omega t < \pi/3$ ⇒ $v_{an} = 0$

For $\pi/3 \le \omega t < 2\pi/3$ ⇒ $v_{an} = v_{AN} = 169.7 \sin \omega t$

$v_{AC} = \sqrt{6}\, Vs \sin(\omega t - \pi/6)$

For $2\pi/3 \le \omega t < \pi$ ⇒ $v_{an} = v_{AC}/2 = -v_{CA}/2 = 147.1 \sin(\omega t - \pi/6)$

For $\pi \le \omega t < 4\pi/3$ ⇒ $v_{an} = v_{AN} = 169.7 \sin \omega t$

For $4\pi/3 \le \omega t < 5\pi/3$ ⇒ $v_{an} = v_{AB}/2 = 147.1 \sin(\omega t + \pi/6)$

For $5\pi/3 \le \omega t < 2\pi$ ⇒ $v_{an} = v_{AN} = 169.7 \sin \omega t$

$v_{AB} = \sqrt{6}\, Vs \sin(\omega t + \pi/6)$